

# Image Restoration using Bregman Iteration

V. Krithika

ME Computer Science and Engineering,  
P.A. College of Engineering and Technology, Pollachi, Tamilnadu, India

**Abstract** - The digital image processing refers to processing of two dimensional images by digital computer. The image restoration is the reconstruction process applied to the degraded images. The removal of noise from images can be done by filters. New techniques are involved in the field of restoration other than filters. Nonlocal Image representation has shown great potential in various low-level vision tasks. The usage of patches introduced many ideas of restoration process. The spatially adaptive iterative single value thresholding algorithm provides better results by using patches. The dictionary used by the algorithm is based on discrete cosine transform or principle component analysis. The patch based restoration suffers from two problems called computational complexity of dictionary learning and inaccurate sparse coding coefficients due to ignorance of relationship among patches. The above two problems are avoided by group based sparse representation of images. The group sparsity relaxes the complexity by having similar patches in the group. The dictionary used is self-adaptive exploiting self-similarity of images. The  $\ell_0$  minimization problem occurred during patch matching will be overcome by Split Bregman iteration.

**Index terms**--- Discrete Cosine Transform, Principle Component Analysis, Split Bregman iteration

## 1. INTRODUCTION

Digital images are electronic snapshots taken of a scene composed of picture elements in a grid formation known as pixels. Each pixel holds a quantized value representing the tone at a specific point. The formation of digital images can be described by the pinhole camera model as a point and only the light from the scene passes through the camera aperture can be captured on the image plane.

The camera aperture is of finite size and the deviation of light reflected from the object will create an image. The process of light falls on object is illumination. Both illumination and reflection is the fundamental process for creation of an image. The image acquisition is the initial step of image processing. The defects occurred during acquisition will lead to image restoration. The effects can be caused through both hardware and software.

The observed or distorted image  $i(x,y)$  can be modeled as a convolution of the object function  $o(x,y)$  in the actual object in the scene and the image degradation function  $h(x,y)$ . Equation.1 is known as the point spread function.

$$i(x,y) = o(x,y) ** h(x,y) + n(x,y) \quad (1)$$

Where  $n(x,y)$  is an additive noise function describing the random variation of the pixel intensity. Figure 1 shows image degradation and restoration process.

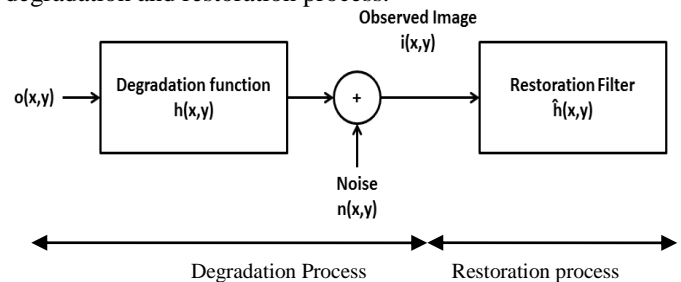


Figure.1 Image degradation and restoration block diagram

The restoration process can be done by using various filters. The filters employed can be linear or anisotropic. The linear filters work by convolving original image with mask. The anisotropic filters work by diffusion done perpendicular to the image. The filters are applied to various noises. The restoration process is quite simple but the actual implementation of filters is difficult. The filters must require degradation function and type of noise to be known in prior.

The restoration can also be done by using non-local means algorithm [1]. The Non-Local means is the averaging of non-local pixels in the image. The NL means algorithm provides better performance than filters. Various approaches of image restoration has been studied in ([1]-[6], [15], [16], [18], [20]).

## 2. RELATED WORK

Stewart [12] has provided the importance of singular value decomposition and its advantages such as stability, low rank approximation of matrices. Decomposition can be achieved by unitary matrices. It can be applied for Spatially Adaptive Iterative Single value Thresholding (SAIST) algorithm [22] in case of minimizing matrix during iterative regularization. Kenneth Rose [13] introduces a novel method called deterministic annealing method for clustering. The method introduces equi-probable contours as cluster regions and it is used for clustering and classification.

Block Matching [3] achieved the sparsity by grouping similar 2D image fragments into 3D data arrays called as groups. Collaborative filtering is a special procedure developed to deal with these 3D groups. The collaborative filtering reveals even the nest details shared by grouped blocks and at the

same time it preserves the essential unique features of each individual block. The new approach of soft decision interpolation [23] for estimating missing pixels in groups preserves spatial coherence of interpolated images better than previous methods and it produces the best results in both PSNR measure and subjective visual quality.

Julien Mairalet et al [4] focused on self-similarities in natural images to average out noise among similar patches and sparse coding encodes natural image statistics by decomposing each image patch into a linear combination of a few elements from the basis set called dictionary. It has been applied only to uniform noise models and it does not applied for image inpainting and texture synthesis. A low rank approach [9] is applied to the problem of matrix approximation. It involves soft thresholding operation on matrices. The convergence problem is occurred due to non-decreasing threshold value.

Osher et al [14] introduces the procedure of iterative regularization procedure for inverse problems based on the use of Bregman distances. The method motivates the problem of restoring noisy and blurry images through variational methods by using total variation regularization. The iteration starts with initial vector and it proceeds until the threshold is reached for spatial adaptation.

Michael R. Charest Jr et al [17] formulated the effective methods of denoising images by using iterative refining cost function. Iterative twicing regularization, Unsharp regularization are used to denoise and deblur images. The second method derived by author provides better results among all methods. The proposed method can also be extended to update the noise variance and signal variance. Jose M. Bioucas-Dias and Mario A.T. Figueiredo [8] developed two step iterative shrinkage thresholding method. The aim is to solve linear inverse problems. The method will consider the result of last two previous iterates than only one iterate. A fast iterative shrinkage thresholding for linear problems and a monotone version of it for nonlinear inverse problems are derived.

A fast gradient algorithm [7] was used to solve deblurring and denoising problems. The algorithm used is gradient projection dual approach method. The iteration used is Fast Iterative Shrinkage Thresholding. It combines with both gradient projection and fast gradient projection. The fast gradient projection scores over gradient projection in solving subproblems of denoising. Jian Zhang [11] developed a new method for solving l0 minimization problem with Bregman iteration for compressive sensing. The basic idea of Split Bregman iteration is to solve the problem of l1 minimization. The methodology is to convert unconstrained problems into constrained one and solve it by Bregman iteration. The result shows good convergence property but it could be applied only for basis pursuit problem.

### 2.1. Block matching 3d filtering

The block transformation [3] is a novel technique used to denoise an image. It involves 3d transformation of image blocks. Initially a reference block is fixed and the similar blocks are obtained by using similarity measure Euclidean distance. The similar blocks are grouped to form 3d blocks. The basic estimate is obtained by using thresholding or by using wiener filtering. Each fragment will produce its own estimate and by weighted averaging, overall estimate is obtained for the block. The final estimate of an image is obtained by aggregating the estimates of all pixels.

The block matching method can be adapted to various noise models such as additive colored noise and non- gaussian noise by modifying the calculation of coefficients variances in the basic and Wiener parts of the algorithm. The developed method can be modified for denoising 1D-signals and video [6] for image restoration and for other problems to benefit from highly sparse signal representations.

### 2.2 SAIST algorithms

The non-local image model exposes self-similarity of an image. The combined approach of sparsity and self-similarity will lead to Simultaneous Sparse Coding (SSC). The SSC technique used basis set called dictionary. The dictionary collects patches from various images. Each patch is of size 9 x 9. The dictionary can be created by using Discrete Cosine transformation or Principle Component Analysis. The Spatially Adaptive Iterative Singular value Thresholding (SAIST) algorithm [21] combines sparsity versus self-similarity by using SSC.

Simultaneous sparse coding (SSC) or group sparsity is one of the leading in image restoration methods. It uses non-local image restoration process. The noisy data will be removed from degraded image by considering variance estimation. The noise and signal variance are considered in each iteration. A low rank approach called singular value decomposition is used for building dictionary at each iteration. The denoising chooses Bayshrink variance estimation and for incomplete data Deterministic annealing method ([19], [13]) by incorporating the idea of dictionary learning is used. The proposed of bilateral variance estimation with dictionary learning has shown greater results in image restoration. The PSNR measure is higher than previous Block Matching method.

#### 2.2.1 Image denoising

It is a technique to remove noise from degraded images. The denoising involves various methods other than filters ([2], [5], [15]) for reconstruction of image. It is done by using dictionary learned from Discrete Cosine Transform (DCT) or Principle Component Analysis (PCA).The degraded image

will be divided into patches. Each patch will be iterated over test image. The similarity measure is calculated for each iteration. Based on the measured Structural Similarity Measure (SSIM) the patches are clustered and undergo thresholding to remove noise. The final image is upgraded by weighted averaging of all similar patches.

### 2.2.2 Image completion

It is done by technique called Inpainting. Inpainting is a kind of technique in image restoration. The user will select the region to be restored and the algorithm will automatically fills-in these regions with information surrounding the inpainting area. The fill-in is done in such a way of arriving isophote lines at the regions boundaries are completed inside. The technique used does not require the user to specify the novel information about the lines. The filling of regions by joining the boundary pixels is done automatically after user specified the region. The inpainting does not impose any other topological information such as Structure of region to be inpainted.

The in painting [16] is completed by using dictionary containing patches. The image to be in painted will undergo iterative regularization with the patches. The threshold value is calculated on each iteration by using Deterministic Annealing (DA). It proceeds until the threshold value selected will complete the image. The image is upgraded finally by updating the dictionary.

### 2.3 Optimization problem

The image restoration involves the problem of l0 optimization due to dictionary with more similar patches. Various methods for solving l0 minimization problem has been studied in ([7], [8], [10]). The l0 optimization can be taken as l1 problem in some technical cases. The l1 optimization problem could be solved by Split Bregman iteration [21]. The l0 optimization has been applied for compressed sensing in [11]. The Bregman iteration could involve optimization of dictionary and solve the problem of minimization. The Split Bregman algorithm is

**Step 1:** Set  $t=0$

**Step 2:** Repeat

**Step 3:**  $u^{(t+1)} = \arg\min_u f(u) = \frac{\mu}{2} \|u - Gv^{(t)} - b^{(t)}\|_2^2$

**Step 4:**  $v^{(t+1)} = \arg\min_v g(v) = \frac{\mu}{2} \|u^{(t+1)} - Gv - b^{(t)}\|_2^2$

**Step 5:**  $b^{(t+1)} = b^{(t)} - (u^{(t+1)} - Gv^{(t+1)})$

**Step 6:**  $t=t+1$

**Step 7:** until stopping criterion is reached.

## 3. PROPOSED WORK

The group sparsity is combined with self-adaptive dictionary reduces the complexity of missing patches in the dictionary. The l0 minimization problem will be solved by Split Bregman iteration.

### 3.1 Image deblurring

The steps involved in image deblurring is

**Step 1:** Blur the image with Gaussian function.

**Step 2:** Decomposition of image into patches to form dictionary.

**Step 3:** The similar patches could form clusters.

**Step 4:** The singular value decomposition is used to learn dictionary.

$$\text{svd}(A) = USV^T \quad (2)$$

where  $A$  = input image

$U$  = rows entries of image patch

$V$  = column entries of image patch

$S$  = diagonal entries containing Eigen vectors.

**Step 4:** Split Bregman iteration of patches onto the image to restore the image.

**Step 5:** The dictionary and sparse coding coefficients updated at the end of final iteration.

**Step 6:** Image updated by using resulted dictionary.

### 3.2 Image inpainting

The image in painting involves the same steps to complete the image. The mask involving diagonal entries are used to complete the image instead of blur operator. The idea is to cover pixels in the neighborhood value to cover the incomplete region.

## 4. RESULTS AND DISCUSSION

The experimental results of Saist algorithm is obtained with higher psnr value than other previous methods. The sample gray images of size 512X512 are taken as input. The iteration count of ten produces denoised image of PSNR 41.11 and SSIM of 0.9761 is obtained for Barbara image. The lena image is denoised with PSNR of 41.00 and SSIM of 0.9622.

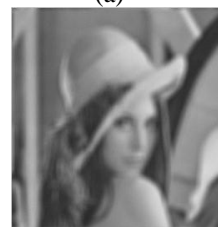
The performance measure is given in Table 1. The deblurring results are shown in Figure.2.



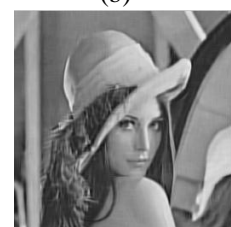
(a)



(b)



(c)



(d)

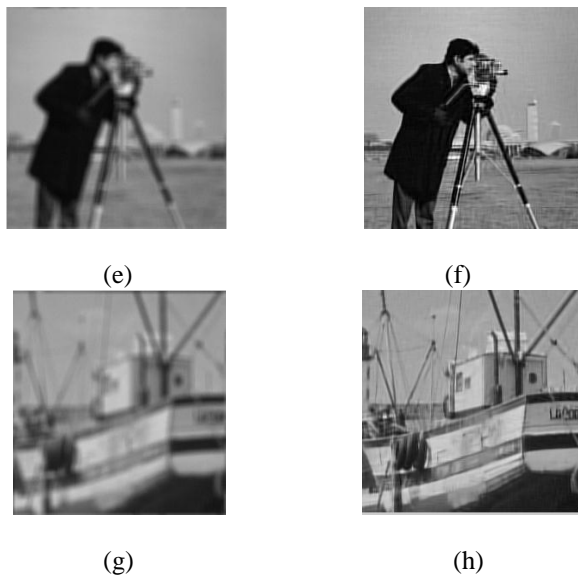


Figure. 2 Results of deblurring

- a) Degraded Barbara image b) Deblurred Barbara image  
c) Degraded Lena image d) Deblurred Lena image  
e) Degraded cameraman image f) Deblurred cameraman image  
g) Degraded boat image h) Deblurred boat image

Image	Original PSNR	Final PSNR
Barbara.tif	25.59	28.05
Lena.png	22.44	30.59
Cameraman.tif	20.75	27.11
Boat.tif	22.27	30.79

Table.1 Performance of group sparsity with Split Bregman iteration

The ten iterations of two images have taken elapsed time of 4.131 min. The choice of choosing similar patch in the dictionary involves more time to restore the image. The action of self-adaptive dictionary eliminates the missing patch condition. The group sparsity involves only few iteration using Split Bregman iteration. It reduces the time complexity of Saist algorithm.

The SAIST algorithm shows denoising results to get converged in 10 iterations as shown in Figure.3. The PSNR value needs more than 3 minutes to restore the image. Figure.4 shows the results of group sparsity with Split Bregman iteration. The new iteration will reduce the time complexity by solving 10 minimization problem. The convergence is obtained within few iterations.

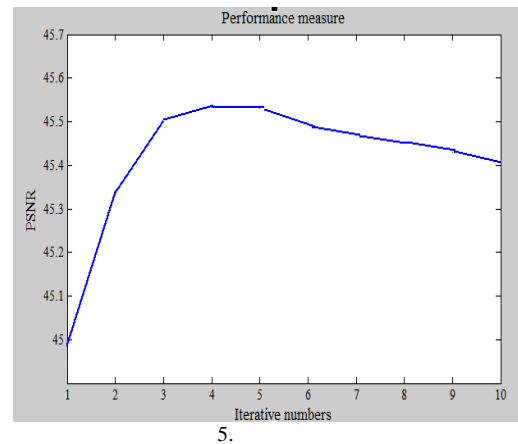


Figure.3 Convergence level of SAIST algorithm

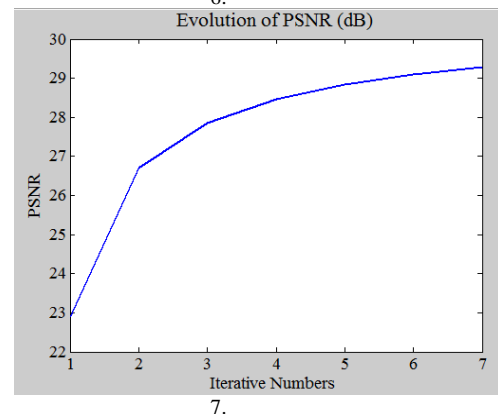


Figure.4 Convergence level of Bregman iteration method

## 5. CONCLUSION

Thus a general framework for high-quality image restoration using group sparsity metho is achieved. The image restoration is done through dictionary of image patches at a time. It explicitly and effectively characterizes local sparsity and nonlocal self-similarity of natural images simultaneously in a unified manner. An effectual self-adaptive group dictionary learning technique with low complexity is designed. High sparsity degree and high recovery quality is achieved by group based sparsity method. It proves its result in the area of image deblurring and inpainting. The PSNR value calculated in the group sparsity method is greater than previous algorithms used in image restoration. The Split Bregman method could solve 10 minimization problem efficiently. The optimization solution will eliminate the issues in time complexity. Thus group sparsity with the self-adaptive dictionary will proves better results in image restoration.

## REFERENCES

- [1] A. Buades, B. Coll, and J.-M. Morel, "A non-local algorithm for image denoising .," in Proc. Conf. Comput. Vis. Pattern Recognit., vol. 2. 2005,pp. 60–65.

- [2] C. Kervrann and J. Boulanger, "Unsupervised patch-based image regularization and representation," in *Proc. Eur. Conf. Comput. Vis.*, 2006, pp. 555–567.
- [3] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, "Image denoising by sparse 3-D transform-domain collaborative filtering," *IEEE Trans. Image Process.*, vol. 16, no. 8, pp. 2080–2095, Aug. 2007.
- [4] J. Mairal, F. Bach, J. Ponce, G. Sapiro, and A. Zisserman, "Non-local sparse models for image restoration," in *Proc. IEEE 12th Int. Conf. Comput. Vis.*, Jun. 2009, pp. 2272–2279.
- [5] P. Chatterjee and P. Milanfar, "Clustering-based denoising with locally learned dictionaries," *IEEE Trans. Image Process.*, vol. 18, no. 7, pp. 1438–1451, Jul. 2009.
- [6] H. Ji, C. Liu, Z. Shen, and Y. Xu, "Robust video denoising using low rank matrix completion," in *Proc. IEEE Conf. Comput. Vis. Pattern Recognit.*, Mar. 2010, pp. 1791–1798.
- [7] A. Beck and M. Teboulle, "Fast gradient-based algorithms for constrained total variation image denoising and deblurring problems," *IEEE Trans. on Image Process.*, vol. 18, no. 11, pp. 2419–2434, Nov. 2009.
- [8] J. Bioucas-Dias and M. Figueiredo, "A new TwIST: two-step iterative shrinkage/thresholding algorithms for image restoration," *IEEE Trans. on Image Process.*, vol. 16, no. 12, pp. 2992–3004, Dec. 2007.
- [9] J. Cai, E. Candes, and Z. Shen, "A singular value thresholding algorithm for matrix completion," *SIAM J. Optim.*, vol. 20, no. 4, pp. 1956–1982, 2010.
- [10] J. Zhang, D. Zhao, R. Xiong, S. Ma, and W. Gao, "Image Restoration Using Joint Statistical Modeling in Space-Transform Domain," *IEEE Transactions on Circuits System and Video Technology* (2014), DOI: 10.1109/TCSVT.2014.2302380.
- [11] J. Zhang, C. Zhao, D. Zhao, and W. Gao, "Image compressive sensing recovery using adaptively learned sparsifying basis via L0 minimization," *Signal Processing* (2013), DOI: 10.1016/j.sigpro.2013.09.025.
- [12] G. Stewart, "On the early history of the singular value decomposition," *SIAM Rev.*, vol. 35, no. 4, pp. 551–566, 1993.
- [13] K. Rose, "Deterministic annealing for clustering, compression, classification, regression, and related optimization problems," *Proc. IEEE*, vol. 86, no. 11, pp. 2210–2239, Nov. 1998.
- [14] S. Osher, M. Burger, D. Goldfarb, J. Xu, and W. Yin, "An iterative regularization method for total variation-based image restoration," *Multiscale Model. Simul.*, vol. 4, no. 2, pp. 460–489, 2005.
- [15] M. Elad and M. Aharon, "Image denoising via sparse and redundant representations over learned dictionaries," *IEEE Trans. Image Process.*, vol. 15, no. 12, pp. 3736–3745, Dec. 2006.
- [16] M. Bertalmio, G. Sapiro, V. Caselles, and C. Ballester, "Image inpainting," in *Proc. SIGGRAPH*, New Orleans, LA, 2000, pp. 417–424.
- [17] M. Charest, M. Elad, and P. Milanfar, "A general iterative regularization framework for image denoising," in *Proc. 40th Annu. Conf. Inf. Sci. Syst.*, 2006, pp. 452–457.
- [18] O. G. Guleryuz, "Nonlinear approximation based image recovery using adaptive sparse reconstructions and iterated denoising-part I: Theory," *IEEE Trans. Image Process.*, vol. 15, no. 3, pp. 539–554, Mar. 2006.
- [19] K. Rose, E. Gurewitz, and G. Fox, "A deterministic annealing approach to clustering," *Pattern Recognit. Lett.*, vol. 11, no. 9, pp. 589–594, 1990.
- [20] O. G. Guleryuz, "Nonlinear approximation based image recovery using adaptive sparse reconstructions and iterated denoising-part II: Adaptive algorithms," *IEEE Trans. Image Process.*, vol. 15, no. 3, pp. 555–571, Mar. 2006.
- [21] T. Goldstein and S. Osher, "The split Bregman algorithm for L1 regularized problems," *SIAM J. Imaging Sci.*, vol. 2, pp. 323–343, Apr. 2009.
- [22] W. Dong, G. Shi, and X. Li, "Nonlocal image restoration with bilateral variance estimation: a low-rank approach," *IEEE Trans. on Image Processing*, vol. 22, no. 2, pp. 700–711, Feb. 2013.
- [23] X. Li and M. Orchard, "New edge directed interpolation," *IEEE Trans. Image Process.*, vol. 10, no. 10, pp. 1521–1527, Oct. 2001.